Hints to Quiz

(1)

Apply the vector formula

$\nabla \cdot (\boldsymbol{a} f) = f \nabla \cdot \boldsymbol{a} + \boldsymbol{a} \cdot \nabla f$,

where $\boldsymbol{a}$ and f correspond to $\nu \nabla A$ and w, respectively, to the integrant.

The boundary term, which appears due to the Gauss divergence theorem, vanishes because of the boundary conditions.

(2) To show that the finite element matrix is sparse, use the fact that ∇N_i is a zero vector in the finite elements which do not contain node i. Consider what happens when nodes i and j are not directly connected by edges.

(3) Taking rotation of $\boldsymbol{E}$ which is expressed in terms of the edge-based interpolation function $\boldsymbol{N}$, we have

$$\mathrm{rot}\boldsymbol{E} = \sum_{I=1}^{\mathrm{e}} e_I \,\mathrm{rot}\, \boldsymbol{N}_I(\boldsymbol{x}), \tag{a}$$

where $e_I = \int_{c_I} \boldsymbol{E} \cdot d\boldsymbol{l}$.

On the other hand, from the Faraday law $\mathrm{rot}\,\boldsymbol{E} = -\partial \boldsymbol{b} / \partial t$, rot$\boldsymbol{E}$ is expected to be expressed in terms of the face-based interpolation function $\boldsymbol{M}$ in such a way that

$$\mathrm{rot}\boldsymbol{E} = \sum_{J=1}^{\mathrm{f}} (\mathrm{rot}\boldsymbol{E})_J \boldsymbol{M}_J = \sum_{J=1}^{\mathrm{f}} \left(\int_{S_J} \mathrm{rot}\boldsymbol{E} \cdot d\boldsymbol{S} \right) \boldsymbol{M}_J = \sum_{J=1}^{\mathrm{f}} \left(\int_{\partial S_J} \boldsymbol{E} \cdot d\boldsymbol{l} \right) \boldsymbol{M}_J . \tag{b}$$

Applying the geometrical formula $\partial S_J = \sum_{I}^{\mathrm{e}} R_{JI} c_I$ to (b) and comparing (b) with (a), you can have the desired equation.

References

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(4)高橋，「三次元有限要素法」，電気学会

(5)Jin, Finite Element Method in Electromagnetics, Wiley